

Exploring a missing link between peculiar, sub- and super-Chandrasekhar type Ia supernovae by modifying Einstein's gravity

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Observations of several peculiar, under- and over-luminous type Ia supernovae (SNeIa) argue for exploding masses widely different from the Chandrasekhar-limit. We explore the modification to Einstein's gravity in white dwarfs for the first time in the literature, which shows that depending on the (density dependent) modified gravity parameter α , chosen for the present purpose of representation, limiting mass of white dwarfs could be significantly sub- as well as super-Chandrasekhar. Hence, this unifies the apparently disjoint classes of SNeIa, establishing the importance of modified Einstein's gravity in white dwarfs. Our discovery questions both the global validity of Einstein's gravity and the uniqueness of Chandrasekhar's limit.

Keywords: Modified gravity; white dwarfs; supernova type Ia

1. Introduction

Since last few years, we have been exploring physics behind peculiar type Ia supernovae (SNeIa), which are highly over-luminous, and the possible existence of super-Chandrasekhar white dwarfs. Our initiation has brought the topic of super-Chandrasekhar white dwarfs in limelight, with so many papers published following us.

SNeIa are believed to be triggered from the violent thermonuclear explosion of a carbon-oxygen white dwarf on approaching its mass the Chandrasekhar limit of $1.44M_{\odot}$ ¹. SNIa is used as a standard candle in understanding the expansion history of the universe². Nevertheless, some of these SNeIa are highly over-luminous, e.g. SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc^{3,4}, and some others are highly under-luminous, e.g. SN 1991bg, SN 1997cn, SN 1998de, SN 1999by^{5,6}. The luminosity of the former group (super-SNeIa) implies the existence of highly super-Chandrasekhar progenitor white dwarfs with mass $2.1 - 2.8M_{\odot}$ ^{3,4}. While, the latter group (sub-SNeIa) predicts the progenitor mass to be as low as $\sim M_{\odot}$ ⁵.

While we argued, in a series of papers, that highly magnetized white dwarfs could be as massive as inferred from the above super-SNeIa observations⁷⁻⁹, they are unable to explain the sub-SNeIa. All the previous models proposed to describe them entail caveats. For example, although numerical simulations of the merger of two sub-Chandrasekhar white dwarfs reproduce the sub-SNeIa, the underlying simulated light-curves fade slower than that suggested by observations.

Nonetheless, a major concern is a large number of models required to explain apparently the same phenomena, i.e., triggering of thermonuclear explosions in white dwarfs. Why there are mutually uncorrelated sub- and super-SNeIa in nature? This is where the proposal of modifying general relativity steps in into the context

2

of white dwarfs. We will show that modified general relativity unifies the sub-classes of SNeIa by a single underlying theory, hence serve as a missing link.

2. Basic equations and formalism

Let us start with the 4-dimensional action as¹⁰

$$S = \int \left[\frac{1}{16\pi} f(R) + \mathcal{L}_M \right] \sqrt{-g} d^4x, \quad (1)$$

where g is the determinant of the spacetime metric $g_{\mu\nu}$, d^4x the 4-dimensional volume element, \mathcal{L}_M the Lagrangian density of the matter field, R the scalar curvature defined as $R = g^{\mu\nu} R_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor and f is an arbitrary function of R ; in general relativity, $f(R) = R$.

For the present purpose, we consider the simplistic Starobinsky model¹¹ defined as $f(R) = R + \alpha R^2$, when α is a constant. However, similar effects could also be obtained in other, physically more sophisticated, theories, where α (or effective- α) is varying (e.g., with density). Now, on extremizing the action Eq. (1) for Starobinsky's model, one obtains the modified field equation of the form

$$G_{\mu\nu} + \alpha \left[2RG_{\mu\nu} + \frac{1}{2}R^2g_{\mu\nu} - 2(\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)R \right] = 8\pi T_{\mu\nu}, \quad (2)$$

where $T_{\mu\nu}$ contains only the matter field (non-magnetic star), $G_{\mu\nu}$ is Einstein's field tensor, ∇_μ and ∇_ν are the covariant derivatives and $\square = \nabla_\mu \nabla^\mu$ (see Refs. 12, 13, for details).

For the present purpose, we seek perturbative solutions of Eq. (2) (see, e.g., Ref. 14), such that $\alpha R \ll 1$. Furthermore, we consider the hydrostatic equilibrium condition so that $g_{\nu r} \nabla_\mu T^{\mu\nu} = 0$, with zero velocity. Hence, we obtain the differential equations for mass $M_\alpha(r)$, pressure $P_\alpha(r)$ (or density $\rho_\alpha(r)$) and gravitational potential $\phi_\alpha(r)$, of spherically symmetric white dwarfs (which is basically the set of *modified* Tolman-Oppenheimer-Volkoff (TOV) equations). For $\alpha = 0$, these equations reduce to TOV equations in general relativity.

As the white dwarf is assumed to be nonmagnetized, we consider EoS, as obtained by Chandrasekhar¹ at extremely low and high densities, $P_0 = K\rho_0^{1+(1/n)}$, where P and ρ of Ref. 1 are replaced by P_0 and ρ_0 respectively ($\alpha = 0$: general relativity) in the spirit of perturbative approach. Here, n is the polytropic index and K a dimensional constant. The boundary conditions are: $M_\alpha(0) = 0$ and $\rho_\alpha(0) = \rho_c$ (central mass and density respectively). Note that by varying ρ_c from 2×10^5 gm/cc to 10^{11} gm/cc, we construct the mass-radius relation of white dwarfs.

3. Results

We show in Figs. 1(a) and (b) that all three $M_\alpha - \rho_c$ curves for $\alpha > 0$ overlap with the $\alpha = 0$ curve in the low density region. However, as α increases, the region of overlap decreases, receding to a lower ρ_c region. At $\rho_c \gtrsim 10^8$, 4×10^7 and

2×10^6 gm/cc, modified general relativity effects become important and visible for $\alpha = 2 \times 10^{13}$ cm², 8×10^{13} cm² and 10^{15} cm² respectively. At a fixed α , with the increase of ρ_c , first M_α increases, then by reaching a maximum value starts decreasing, like the $\alpha = 0$ (general relativity) case. The maximum mass M_{\max} decreases with the increasing α and for $\alpha = 10^{15}$ cm² it is as low as $0.81M_\odot$ (highly sub-Chandrasekhar). This argues that modified general relativity has a tremendous impact on white dwarfs. In fact, $0.81 \lesssim M_{\max}/M_\odot \lesssim 1.31$ for all the chosen $\alpha > 0$. This is a remarkable finding since it establishes that even if ρ_c s for these sub-Chandrasekhar white dwarfs are lower than the conventional value at which SNeIa are usually triggered, an attempt to increase the mass beyond M_{\max} with the increase of ρ_c will lead to a gravitational instability. Subsequently, this presumably will be leading to a runaway thermonuclear reaction, provided the core temperature increases sufficiently due to collapse. Occurrence of such thermonuclear runaway reactions, triggered at a low density as 10^6 gm/cc, has already been demonstrated¹⁵. Thus, once M_{\max} is approached for white dwarfs with $\alpha > 0$, a SNIa is expected to trigger just like in the $\alpha = 0$ case, explaining the sub-SNeIa^{5,6}, like SN 1991bg mentioned above.

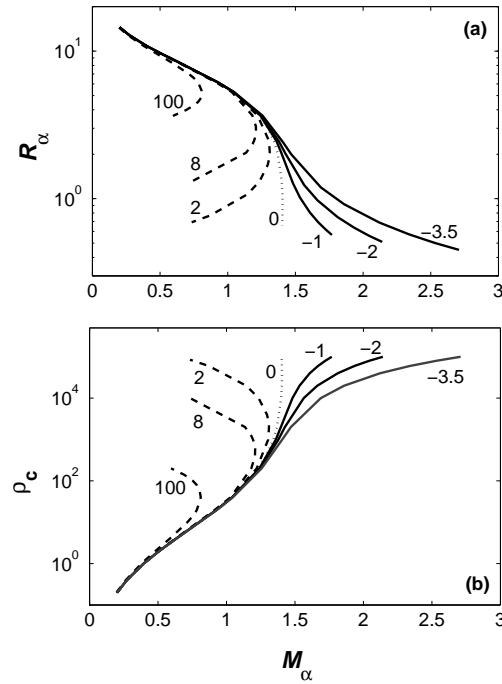


Fig. 1. Unification of under-luminous and over-luminous SNeIa: (a) mass-radius relations, (b) variation of ρ_c with M_α . The numbers adjacent to the various lines denote $\alpha/(10^{13} \text{ cm}^2)$. ρ_c , M_α and R_α are in units of 10^6 gm/cc, M_\odot and 1000 km respectively.

Figure 1(b) shows that for $\rho_c > 10^8$ gm/cc with $\alpha < 0$, the $M_\alpha - \rho_c$ curves deviate from the general relativity curve due to modified general relativity effects. Note that M_{\max} for all the three cases corresponds to $\rho_c = 10^{11}$ gm/cc, what upper-limit is chosen to avoid possible neutron-drip. Interestingly, all values of M_{\max} , lying in $1.8 - 2.7M_\odot$, are highly super-Chandrasekhar. Thus, while the general relativity effect is very small (but non-negligible), modified general relativity effect could lead to $\sim 100\%$ increase in the limiting mass. The corresponding values of ρ_c are large enough, i.e. larger than ρ_c corresponding to M_{\max} of $\alpha = 0$ case, to initiate thermonuclear reactions, whereas the respective core temperatures are expected to be similar. This explains the entire range of the observed super-SNeIa mentioned above^{3,4}, assuming the furthermore gaining mass above M_{\max} leads to SNeIa.

Table 1 ensures the validity of perturbation approach of the solutions, where we solve the modified TOV equations only up to $\mathcal{O}(\alpha)$. Since the product αR is first order in α , we replace R in it by the zero-th order Ricci scalar $R^{(0)} = 8\pi(\rho^{(0)} - 3P^{(0)})$, i.e. Ricci scalar in general relativity ($\alpha = 0$). For the perturbative validity of the entire solution, $|\alpha R^{(0)}|_{\max} \ll 1$ should satisfy. Next, we consider $g_{tt}^{(0)}/g_{tt}$ and $g_{rr}^{(0)}/g_{rr}$ (ratios of $g_{\mu\nu}$ -s in general relativity and those in modified general relativity up to $\mathcal{O}(\alpha)$), which should be close to unity for the validity of perturbative method¹⁶. Hence, $|1 - g_{tt}^{(0)}/g_{tt}|_{\max} \ll 1$ and $|1 - g_{rr}^{(0)}/g_{rr}|_{\max} \ll 1$ should both satisfy. Table 1 indeed shows that all three measures quantifying the validity of perturbative are at least 2 – 3 orders of magnitude smaller than 1.

Table 1. Measure of validity of perturbative solutions corresponding to M_{\max} in Fig. 1.

$\alpha/(10^{13} \text{ cm}^2)$	$ \alpha R^{(0)} _{\max}$	$ 1 - g_{tt}^{(0)}/g_{tt} _{\max}$	$ 1 - g_{rr}^{(0)}/g_{rr} _{\max}$
2	7.4×10^{-5}	6.8×10^{-5}	2.0×10^{-4}
8	7.4×10^{-5}	6.8×10^{-5}	2.0×10^{-4}
100	7.4×10^{-5}	6.9×10^{-5}	2.0×10^{-4}
-1	0.00184	0.0016	0.0052
-2	0.00369	0.0031	0.0108
-3.5	0.00646	0.0052	0.0199

4. Possible chameleon-like effect for density dependent model parameter

we now justify that the effects of modified general relativity based on a more sophisticated calculation, invoking an (effective) α varying explicitly with density (and effectively becoming negative), are likely to converge to those described above. Note that even though α is assumed to be constant within individual white dwarfs here, there is indeed an implicit dependence of α on ρ_c , clearly shown in Fig. 1(b) for limiting mass white dwarfs presumably leading to SNeIa. This indicates the existence of an underlying chameleon effect, which trend is expected to emerge self-consistently

in a varying- α theory.

Let us consider a possible situation where α is varying explicitly with density and try to relate it with the results presented above. Note the fact that the super-SNeIa occur mostly in young stellar populations consisting of massive stars (see, e.g., Ref. 3), while the sub-SNeIa occur in old stellar populations consisting of low mass stars (see, e.g., Ref. 17). The massive stars with higher densities are likely to give rise to super-Chandrasekhar white dwarfs on collapse, which, on gaining mass, would subsequently explode to produce super-SNeIa. The low mass stars with lower densities would be expected to give rise to sub-Chandrasekhar white dwarfs on collapse, which furthermore would probably end with sub-SNeIa. Now, let us assume α to be depending on density in such a way that there are two terms — one with negative sign dominates at higher densities and the other with positive sign dominates at lower densities. Hence, when a massive, high density star collapses, it results in similar to our $\alpha < 0$ cases; while a low mass, low density star collapse leads to results like our $\alpha > 0$ cases. Thus, the same functional form of α could lead to both super- and sub-Chandrasekhar limiting mass white dwarfs, respectively, depending on their densities. Of course, the final mass of the white dwarf would depend on several factors, such as, ρ_c and the density gradient in the parent star, etc. Interestingly, this description of density dependent α is essentially equivalent to invoking a so-called “chameleon- $f(R)$ theory”, which can pass solar system tests of gravity (see, e.g., Ref. 18). This is so because, once α is a function of density, it is a function of R . Hence, introduction of a density (and hence R) dependent α is equivalent of choosing an appropriate (more complicated) $f(R)$ model of gravity. Therefore, a more self-consistent variation of α with density does not invalidate the results of the constant- α cases, rather is expected to complement the picture.

We must mention that the orders of magnitude of α are different between typical white dwarfs ($\alpha_{\text{WD}} \sim 10^{13} \text{ cm}^2$, as used above) and neutron stars ($\alpha_{\text{NS}} \sim 10^9 \text{ cm}^2$, e.g. ^{14,19}). This again argues for the fact that there is an underlying chameleon effect which causes α to be different in different density regimes. Now, the quantity αR would have a similar value in both neutron stars and white dwarfs in the perturbative regime. Hence, due to their higher curvature and density, neutron stars will harbor a smaller value of α compared to white dwarfs. Roughly, neutron stars are 10^4 times denser than white dwarfs and, therefore, α_{NS} is 10^4 times smaller than α_{WD} . We also emphasize that the current work is an initiation of the exploration of the effects of modified gravity in white dwarfs, based on the motivation to explain observations of peculiar SNeIa. Now, one has to polish the model step by step.

5. Summary

Based on a specific type of modified Einstein’s gravity, namely simple Starobinsky $f(R)$ -model, we show that modifications to general relativity are indispensable in white dwarfs, in particular to explain observed data related to their limiting mass. It remarkably explains and unifies a wide range of SNeIa for which general relativ-

ity is insufficient. Although the present study is based perturbative method, this is indeed useful as then we have a handle on α characterizing our model, which has an upper bound from astrophysical observations²⁰. Hence, depending on the magnitude and sign of α , we obtain both highly super-Chandrasekhar and highly sub-Chandrasekhar limiting mass white dwarfs, which furthermore help to establish them as progenitors of the peculiar super- and sub-SNeIa, respectively. Thus, a single underlying theory, i.e. an $f(R)$ -model, unifies the two apparently, puzzling, disjoint sub-classes of SNeIa, hence serves as a missing link. Our discovery raises two fundamental questions. Is the Chandrasekhar limit unique? Is Einsteins gravity the ultimate theory for understanding astronomical phenomena? Both the answers appear to be no!

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